

ELASTIC MESON-NUCLEON AND NUCLEON-NUCLEON SCATTERING: MODELS vs. ALL AVAILABLE DATA ^a

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We consider simple-pole descriptions of soft elastic scattering at small t and s , and allow for the presence of a hard pomeron. We have built and analyzed an exhaustive dataset and we show that simple poles provide an excellent description of the data in the region $-0.5 \text{ GeV}^2 < t < -0.1 \text{ GeV}^2$, $6 \text{ GeV} < \sqrt{s} < 63 \text{ GeV}$. We show that new form factors have to be used, and get information on the trajectories of the soft and hard pomerons.

Recently ¹, we have shown that a model that includes a hard pomeron reproduces very well (with $\chi^2/d.o.f. \approx 0.95$ or with confidence level CL=93%) total cross sections and the ratio ρ of the real to imaginary parts of the forward scattering amplitude, while the description obtained from a soft pomeron is much less convincing ² ($\chi^2/d.o.f. \approx 1.07$, CL=6%). We considered the full, standard set of forward data ³ for pp , $\bar{p}p$, Kp , πp , γp and $\gamma\gamma$, and showed that the description extends down to $\sqrt{s} = 5 \text{ GeV}$.

The details of these results are presented in the contribution of J.R. Cudell to these proceedings ⁴. However, despite an excellent χ^2 and the fact that the hard pomeron intercept is very close to what is observed in deeply inelastic scattering ⁵ and in photoproduction ⁶, it is not entirely sure that it is present in soft scattering. Indeed, its couplings are small and its contribution is less than 10% for $\sqrt{s} < 100 \text{ GeV}$. Hence it is important to look for confirmation of its presence in other soft processes, and the obvious place to start from is elastic scattering. This analysis is briefly presented in this paper.

1. Scattering amplitudes. We parametrise all exchanges by simple poles, and limit ourselves to a region in s and t where these are dominant. The normalization of the amplitude $A^{ab}(s, t)$ that describes the elastic scattering of hadrons a and b is given by

$$\sigma_{tot}^{ab}(s) = \frac{1}{2q_{ab}\sqrt{s}} \Im m A^{ab}(s, 0), \quad \frac{d\sigma_{el}^{ab}(s, t)}{dt} = \frac{1}{64\pi s q_{ab}^2} |A^{ab}(s, t)|^2, \quad (1)$$

where $q_{ab} = \sqrt{[s_{ab}^2 - 4m_a^2 m_b^2]}/4s$ ($s_{ab} = s - m_a^2 - m_b^2$) is the momentum of particles a and b in the center-of-mass system.

Regge theory implies that one can write $A(s, t) \equiv A(z_t, t)$, where the Regge variable, $z_t = (t + 2s_{ab})/\sqrt{(4m_a^2 - t)(4m_b^2 - t)}$ is the cosine of the scattering angle in the crossed channel. Absorbing in the t -dependent factors the coupling functions of the standard contribution of a Regge pole, one can write for the case of scattering of a on protons

$$A_R^{ap}(\tilde{s}_{ap}, t) = \eta_C g_R^a F_R^a(t) F_R^p(t) (-i\tilde{s}_{ap})^{\alpha_R(t)}. \quad (2)$$

with $F_a^R(0) = 1$, $a = p, \pi, K$ and where $\tilde{s}_{ab} = (t + 2s_{ab})/s_0$, $s_0 = 1 \text{ GeV}^2$. For a crossing-even, $C = +1$, (resp. crossing-odd, $C = -1$) reggeon $\eta_C = -1$ (resp. i).

The model that we are considering can be written:

$$A^{ap}(s, t) = A_+^{ap}(\tilde{s}_{ab}, t) + A_S^{ap}(\tilde{s}_{ab}, t) + A_H^{ap}(\tilde{s}_{ab}, t) \mp A_-^{ap}(\tilde{s}_{ap}, t) \quad (3)$$

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with the $-$ sign for the (positively charged) particles.

2. The data. Most of the measurements, made throughout the last 40 years, have been communicated to the HEPDATA group⁷, so that one does not need to re-encode all the data. However, some basic work still needs to be done, as there are 80 papers, with different conventions, and various units. The global dataset contains 10188 points (at $t \neq 0$). Our detailed analysis of these data and their systematic errors, list of the subsets (measurements at different energies and momentum transfers) will be given in a forthcoming paper⁸. The statistics of the data used in the present analysis is given in Table 1. A few subsets (5% of whole set) were eventually excluded from the fits because they strongly contradict⁸ the main part of the data (see section 5).

Table 1: The statistics of the full dataset and of the present analysis.

observable	N_{pp}	$N_{\bar{p}p}$	N_{π^+p}	N_{π^-p}	N_{K^+p}	N_{K^-p}	N_{tot}
$d\sigma_{el}/dt$ (full set)	4639	1252	802	2169	595	731	10188
this analysis	818	281	290	483	166	169	2207
after exclusion	795	226	281	478	166	169	2115

3. Local fits. In order to obtain the possible form factors, we scan the dataset at fixed t , *i.e.* we fit small windows in t to a complex amplitude with constant form factors (and refer to these fits as *local* fits). The constants that we get will then depend on t and give us a picture of the form factor. The value of the χ^2 will also tell us in which region of t we should work.

This strategy however will not work for the general case considered here: each bin does not contain enough points to have a unique minimum. We can take advantage of the fact that both models considered here give comparable intercepts for the crossing-odd and crossing-even reggeon contributions. We can also read off the slopes from a Chew-Frautschi plot. This gives the following f/a_2 and ρ/ω trajectories:

$$\alpha_+ = 0.61 + 0.82 t, \quad \alpha_- = 0.47 + 0.91 t \quad (4)$$

Furthermore, we shall not be able to include a hard pomeron in the local fits as its contribution is too small to be stable.

We fit the data from $6 \text{ GeV} \leq \sqrt{s} \leq 63 \text{ GeV}$, and we choose small bins of width 0.02 GeV^2 . We correct for the bin width by assuming that within the bin the differential cross section can be approximated by an exponential. We restrict ourselves to independent bins where we have more than four points for each process.

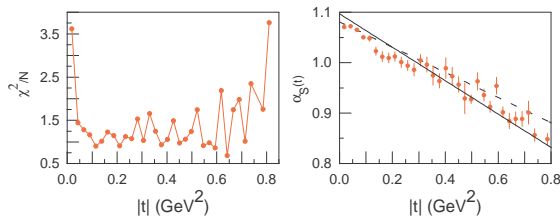


Fig.1 The results of the local fits for the χ^2 per number of points (left) and for the pomeron trajectory (right). The dashed curve is from⁹ and the solid curve results from the global fit given in the section 5.

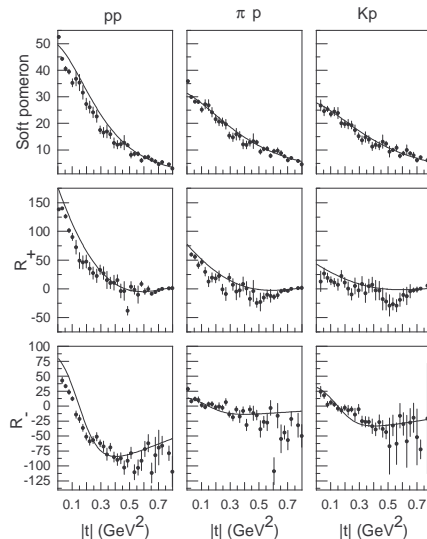


Fig.2 The results of the local fits for the residues of the poles. The curves are the results of a global fit explained in the section 5.

Each of these fits gives us a values of the χ^2 per number of points, $\alpha_S(t)$ for each t , as well as the coefficients $g_R^{ap} F_R^p(t) F_R^a(t)$. The results given in Figs. 1 and 2 show two things: 1) the local fit to all data is never perfect, this can be traced back to incompatibilities in the data ^b; 2) the simple-pole description of the data has a chance to succeed in a limited region: the χ^2 grows fast both at low $|t|$ and for $|t| > 0.6$. To be conservative, we shall consider a global fit in the region $0.1 \leq |t| \leq 0.5$.

The right-hand graph in Fig. 1 shows the soft pomeron trajectory. It is very linear as a function of t . Its intercept and slope are somewhat different from the standard ones ⁹.

4. Form factors. Figure 2 shows the results for the residues of the poles $g^a F_R^a(t) F_R^p(t)$. In all cases, it is obvious that form factors must be different for different trajectories.

We find that we can get a good description if we take ($a = \pi, K$)

$$F_S^p(t) = \left[1 - t/t_S^{(1)} + \left(t/t_S^{(2)} \right)^2 \right]^{-1}, \quad F_H^p(t) = (1 - t/t_H)^{-2}, \quad F_S^a(t) = F_H^a(t) = (1 - t/t_S^a)^{-1}. \quad (5)$$

In the pp and $p\bar{p}$ cases, other dipoles are necessary to describe the $C = \pm 1$ exchanges:

$$F_C^p(t) = (1 - t/t_C)^{-2}. \quad (6)$$

In the crossing-odd case, the presence of a zero in form factors is certain (see Fig. 2): this is the well-known cross-over phenomenon ¹⁰: the curves for $d\sigma/dt$ for pa and $p\bar{a}$ cross each other at some value of t . This phenomenon is present at several energies, and seems to exist for all processes, for which the cross-over point is at $|t| \approx 0.1 - 0.2 \text{ GeV}^2$ ^c. A similar zero (but less pronounced) is seen for crossing-even form factors. Taking into account these facts we thus parametrise the $C = \pm 1$ reggeons ($a = p, \pi, K$)

$$A_C^{ap}(\tilde{s}_{ap}, t) = \eta_C Z_C^a(t) g_C^a F_C^a(t) F_C^p(t) (-i\tilde{s}_{ap})^{\alpha_C(t)} \quad \text{with} \quad Z_C^a(t) = \frac{\tanh(1 + t/\zeta_C)}{\tanh(1)}. \quad (7)$$

Thus, the factor $Z_C^a(t)$ has a common zero ζ_C , independent of s for p, π, K , but a different one for $C = +1$ and $C = -1$ reggeons. The given form of Z restricts its fast growth with t .

5. Global fits.

Equipped with the information from the local fits, we can now perform a global fit to the elastic data for $0.1 \text{ GeV}^2 \leq |t| \leq 0.5 \text{ GeV}^2$, for $6 \text{ GeV} \leq \sqrt{s} \leq 63 \text{ GeV}$. Fitting with the full data set we obtained $\chi^2/d.o.f. \approx 1.45$ for the model with only a soft pomeron (S) and ≈ 1.33 for the model with soft and hard pomerons (S+H). Excluding the subsets which contradict other data (only 92 points) we obtained the results shown in the Table 2. The values of fitted parameters will be given in ⁸. Here we give only the intercepts and slopes of pomeron trajectories.

Table 2: Differential cross sections: partial values of χ^2 .

Quantity	Number of points	S	S+H
		χ^2/N	χ^2/N
$d\sigma^{pp}/dt$	N=795	0.90	0.86
$d\sigma^{p\bar{p}}/dt$	226	1.01	0.99
$d\sigma^{\pi^+p}/dt$	281	0.90	0.89
$d\sigma^{\pi^-p}/dt$	478	1.18	1.18
$d\sigma^{K^+p}/dt$	166	1.02	1.11
$d\sigma^{K^-p}/dt$	169	1.18	1.12
Totally	2115	1.022	0.997

In the model with only a soft pomeron $\alpha_S(t) = 1.0927 + 0.332t$. If a hard pomeron is included then $\alpha_S(t) = 1.0728 + 0.297t$ and $\alpha_H(t) = 1.45 + 0.1(\pm 0.2)t$. In both cases the slope of the soft pomeron trajectory is higher than the standard one ⁹.

Concluding remarks. We have elaborated a complete dataset, including an evaluation of the systematic errors for all data.

^bThe inclusion of data for $\sqrt{s} \leq 6 \text{ GeV}$ would only make this problem worse.

^cWe tried obtaining these zeros through the rescattering of various trajectories. However, their position and existence then depend strongly on s .

We showed that different reggeons must have different form factors. We confirm that crossing-odd meson exchange has a zero. We also found evidence for a sharp suppression of the crossing-even form factor around $|t| = 0.5 \text{ GeV}^2$.

Because of the quality of the soft pomeron fit, the elastic data do not confirm strongly the need for a hard pomeron. However it is remarkable that the hard pomeron fit gives 0.1 GeV^{-2} for the central value of the slope, in agreement with⁵.

Both models considered there lead to a very good fit that extends well to $S\bar{p}p$ S and Tevatron energies.

We hope that the complete dataset constructed here will serve as a starting point for precise studies of the whole range of elastic scattering and for the comparison of various models.

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